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Evaporative cooling of water in a mechanical draft cooling tower

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Abstract

A new mathematical model of a mechanical draft cooling tower performance has been developed. The model represents a boundary-value problem for a system of ordinary differential equations, describing a change in the droplets velocity, its radii and temperature, and also a change in the temperature and density of the water vapor in a mist air in a cooling tower. The model describes available experimental data with an accuracy of about 3%. For the first time, our mathematical model takes into account the radii distribution function of water droplets.

Simulation based on our model allows one to calculate contributions of various physical parameters on the processes of heat and mass transfer between water droplets and damp air, to take into account the cooling tower design parameters and the influence of atmospheric conditions on the thermal efficiency of the tower. The explanation of the influence of atmospheric pressure on the cooling tower performance has been obtained for the first time.

It was shown that the average cube of the droplet radius practically determines thermal efficiency. The relative accuracy of well-defined monodisperse approximation is about several percent of heat efficiency of the cooling tower. A mathematical model of a control system of the mechanical draft cooling tower is suggested and numerically investigated. This control system permits one to optimize the performance of the mechanical draft cooling tower under changing atmospheric conditions.

1. Introduction

Mechanical draft cooling towers are widely used in industry for deep cooling of circulating water [1–3]. The height of the mechanical draft cooling towers can vary from 2 to 12 m. Basic elements of such cooling towers are shown in Fig. 1: the shell, water distribution system, water-collecting pond and the fan to create an artificial draft.

In this paper we consider the heat and mass transfer processes in a mechanical draft cooling tower, where only a water droplet flow takes place, and there are no jet or film flows.

In our previous publication, we considered only monodisperse ensemble of droplets [4]. In the cooling tower a polydisperse ensemble of droplets is formed by nozzles, which spray water. In this work, the size distribution of droplets and elements of two-dimensional aerodynamics of a mechanical draft cooling tower are taken into account. This allows one to explain a variety of experimental data.

In [5], a mathematical model of the cooling tower performance is presented, which allows calculation of the two-dimensional internal aerodynamics of the cooling tower. However, the description of the kinetics of phase transition during evaporative cooling is simplified. The modern level of the description of these processes is presented in [6].

Although in industry the cooling towers with rather wide distribution of droplet radii are used, in the vast majority of simulations of evaporative cooling the approximation of monodisperse ensemble of droplets is used. Our new approach allows one to determine the limits of applicability of this approximation for the problems of evaporative cooling. It is worth to note, that the

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Nomenclature

C	coefficient of aerodynamic drag, dimensionless
c	specific heat ($J\,kg^{-1}\,K^{-1}$)
D	diffusion coefficient for water vapor (m^2/s)
Dt	dispersion of temperature (K^2)
F	function
f	distribution function
g	free fall acceleration (m/s^2)
H	height (m)
h	height of air window (m)
L	cooling tower width (m)
m	mass of droplet (kg)
N	number of groups, dimensionless
N_d	the number of droplets per unit volume (m^{-3})
P	atmospheric pressure (Pa)
Q	air mass flow rate (kg/s)
Q_a	specific air mass flow rate ($kg/m^2\,s$)
Q_w	specific water mass flow rate ($kg/m^2\,s$)
R	droplet radius (m)
r	latent heat of vaporization ($kJ\,kg^{-1}$)
S	informational entropy, dimensionless
S_1	cross-section area of outlet (m^2)
T	temperature (°C)
T_1	required temperature (°C)
u	velocity of air (m/s)
W	power per unit cross-section (kg/s^3)
x	coordinate in horizontal direction
z	coordinate in vertical direction
Z	normalization constant
Nu	Nusselt number
Re	Reynolds number

Greek symbols

α	heat transfer coefficient ($W\,m^{-2}\,K^{-1}$)
β	Lagrange multiplier
Δ	difference
δ	Lagrange multiplier
γ	mass transfer coefficient ($m\,s^{-1}$)
η	thermal efficiency of a cooling tower, dimensionless
λ_a	thermal conductivity of air, (W/m °C)
μ_a	dynamic viscosity of air ($kg\,m^{-1}\,s^{-1}$)
ρ	water vapor density (kg/m^3)
ρ_a	air density (kg/m^3)
ρ_w	water density (kg/m^3)
σ	surface tension of water ($kg\,s^{-2}$)
τ_d	relaxation time of droplet temperature (s)
v	droplet velocity in z direction (m/s)
v_0	droplet velocity in z direction in stagnant air (m/s)
v_1	droplet velocity in z direction in upward air (m/s)
ψ	relative humidity of air, dimensionless
Σ	residual, dimensionless

Subscripts

0	initial
a	air
d	for droplet
f	final
i	index
lim	limiting
s	saturated
w	water
theor	theoretical
exper	experimental
$\langle \cdots \rangle$	mean

pilot model of the microcooling tower with monodisperse droplet distribution has recently been created [7].

For simulation of evaporative cooling of water in the mechanical draft cooling tower, we use the results obtained for a natural draft cooling tower [8]. The solution of a boundary-value problem for two phases moving in the opposite directions gives a complete description of evaporative cooling of droplets. As droplets fall down, the water evaporates and convective heat transfer with a colder air occurs. With increase in the velocity of droplets, the time of interaction with a "fresh", colder air is reduced. On the other hand, as air ascends it is heated and saturated with water vapor. This reduces the intensity of heat and mass transfer of droplets during evaporative cooling.

In a one-dimensional approximation, the average air velocity u is considered constant over the height and section. The air flow velocity u is determined by the fan power and the total aerodynamic drag. In contrast to natural draft cooling towers, where the velocity of convection depends on the degree of air heating and its saturation of it with water vapor.

In a cooling tower the processes of heat and mass transfer depend on the specific mass flow rates of water Q_w and air Q_a, temperature T_{a0} and relative humidity ψ of the air entering into the cooling tower, temperature of the water T_{w0} entering into the cooling tower, wind velocity and atmospheric pressure [9].

We characterize the efficiency of evaporative cooling by means of the dimensionless parameter η [1]:

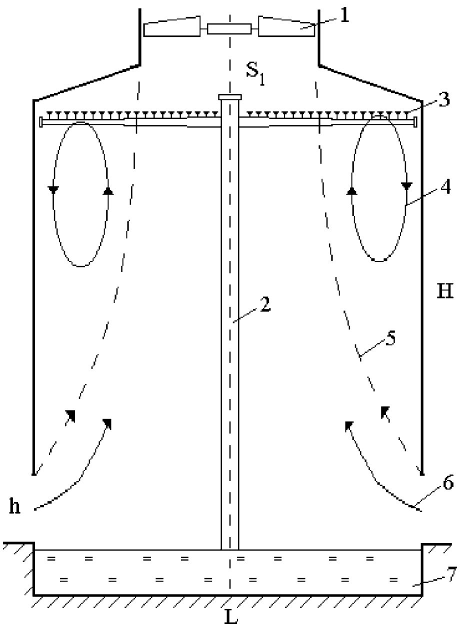

Fig. 1. Scheme of the mechanical draft cooling tower: (1) fan, (2) pipeline, (3) water distribution system with spraying nozzles, (4) stagnant zone, (5) edge of stagnant zone, (6) entering air and (7) water-collecting pond.

$$\eta = \frac{T_{w0} - T_{wf}}{T_{w0} - T_{lim}}, \tag{1}$$

where T_{w0} is the temperature of hot water entering into the cooling tower, T_{wf} is the average temperature of the cooled water in the pool of the cooling tower, T_{lim} is the limiting temperature of evaporative cooling of water for the given air temperature T_a and its relative humidity ψ. The value of T_{lim} is equal to the wet-bulb temperature and is obtained from the condition

$$\rho_s(T_a) \cdot \psi = \rho_s(T_{lim}), \tag{2}$$

where ρ_s is the density of saturated vapor, dependent on temperature and T_a is the temperature of the neighboring cooling tower air.

We note that in the case of monodisperse distribution of droplets the average water temperature T_{wf} coincides with the final temperature of the droplets. When the size distribution of droplets and nonuniformity of air flow in the cooling tower are taken into account, the calculation of the water average temperature in the pool is rather a complex problem.

2. Elements of internal aerodynamics of the mechanical draft cooling tower

Averaged air velocity u, calculated according to the continuity equation in the integral form, is:

$$u = Q/(\rho_a L^2), \tag{3}$$

where Q is the air mass flow rate through the cooling tower defined by the fan and dependent on the air density ρ_a; L is the length of the cooling tower side (assuming that a cooling tower has the square cross-section).

The velocity of the air leaving the cooling tower u_f is

$$u_f = Q/(\rho_a S_1), \tag{4}$$

where S_1 is the area of the cooling tower exit. Since $S_1 < L^2$, it follows from (3) and (4) that the air velocity distribution in the cooling tower is nonuniform. Moreover, at the inlet to the air window of the cooling tower a vertical velocity of air is practically equal to zero, whereas the average inlet horizontal velocity v of air is equal to

$$v = u\frac{L}{4h}, \tag{5}$$

where h is the height of air windows.

As is shown in Fig. 1, in a mechanical draft cooling tower air turns similarly to the turn in an evaporative cooling tower [10]. Stagnant zones with vortex structures are expected because of the narrowing of the air flow at the top of the cooling tower. Some geometric sizes of such a vortex structure can be estimated from the picture of streamlines in Fig. 1.

In the approximation of an incompressible medium, we use the continuity equation [11]

$$\frac{\partial v(x,z)}{\partial x} + \frac{\partial u(x,z)}{\partial z} = 0. \tag{6}$$

Substituting-averaged values of air velocity components in (6), we have the following estimation for the relationship between the cooling tower height H and its width L [12]:

$$\frac{H}{L} \approx 2\frac{4hL}{S_1}. \tag{7}$$

Eq. (7) establishes the relationship between basic geometric parameters of a mechanical draft cooling tower. Usually in engineering practice, the total area of air windows is equal to the area of the cooling tower exit orifice. It follows from Eq. (7) that to attain good aerodynamic quality the cooling tower height should be approximately twice larger than its width.

It is obvious that the large-scale nonuniformity of air velocity distribution causes nonuniform cooling of water droplets in a cooling tower. Consideration of this effect requires the development of a two-dimensional aerodynamic model of cooling tower performance.

To obtain a more precise characteristic of the air velocity distribution in a mechanical draft cooling tower, we introduce the velocity $u(z)$ averaged over the cone cross-section inside the cooling tower, where z is the vertical distance from the fan level. Thus, we can estimate the influence of the nonuniform flow structure on the efficiency of evaporative cooling. Using the continuity equation after simple geometric calculations, we get the following expression for the air flow velocity:

$$u(z) \cong u_f \frac{\pi[H-h]^2}{2[H-h+z(L\sqrt{\pi/2S_1}-1)]^2}. \quad (8)$$

As it follows from Eq. (8), the air velocity decreases with increase in the distance from the fan. The results of the simulation of evaporative cooling of droplets in this field of air velocity will be discussed below. It is obvious that for droplets of different radii the influence of the nonuniformity of the air velocity field should differ significantly.

3. Mathematical model of evaporative cooling of droplets

For a mathematical model of evaporative cooling of droplets, it is important to know the distribution of the radii of the droplets. In a mechanical draft cooling tower droplets are formed by water spraying nozzles. The radii of droplets depend on the water flow rate and water temperature in the cooling tower: the larger the water flow rate, the smaller sizes of droplets because of the higher pressure drop in the nozzles. The water temperature affects the surface tension, which substantially determines the character of water spraying. Our calculations show that the dependence of the radius of droplets on hydraulic loading is determined by design features of the spraying nozzle and is not connected with breaking of droplets. Even at the maximal hydraulic loading of cooling tower the droplets velocities at nozzle exit is insufficient for their breaking.

In a counter-current cooling tower of any type the maximum and minimum radii of droplets are determined, correspondently, by splitting of large droplets and carrying away of small droplets by an air flow. The maximum radius of the droplet falling with the relative velocity v in a humid air flow is determined from the condition of equality of the drag force and surface tension force. Droplets with radius R are not broken, if the following inequality is valid [13]:

$$R \leqslant 2.3\frac{\sigma}{\rho_a v^2}, \quad (9)$$

where σ is the surface tension of water. We note that with increase in the temperature the surface tension of water decreases. This effect must be taken into account in studies of evaporative cooling [8].

The minimal size of the droplets participating in the process of evaporative cooling depends on an upward air flow velocity u. If the drag force due to relative motion of a droplet and air is larger than the gravity, which is valid for rather small droplets, they are carried away by the ascending air flow.

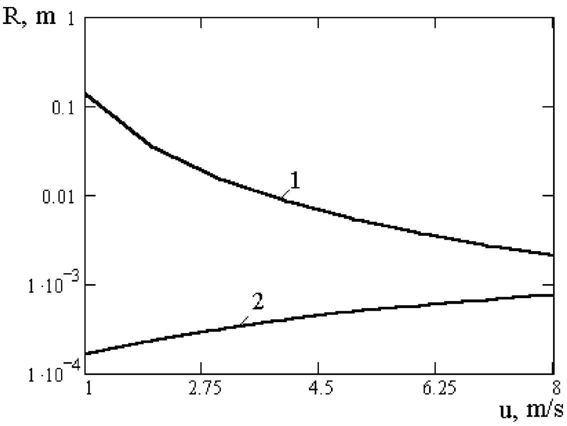

Fig. 2. Possible range of droplets radii versus upward flow air velocity: curve 1 is for maximum possible droplets radius and curve 2 is for minimum.

In Fig. 2 the range of possible radii of water droplets falling in a mechanical draft cooling tower is shown versus the upward air flow velocity. Curve 1 corresponds to the largest possible radii of droplets, which are found from Eq. (9), and curve 2 is for minimally possible radii. As is seen from Fig. 2, in the mechanical draft cooling tower the droplets have the radii with values between curves 1 and 2. If the radius of a droplet is in the area above curve 1, such a droplet is broken up by air flow; the droplet is carried away by upward air flow from the cooling tower if the radius of the droplet is in the area below curve 2.

In the case of a polydisperse ensemble of droplets, we have to deal with some size distribution function of droplets (Fig. 3). However, such an approach makes the mathematical description much more complex, therefore, to simplify the problem, we use the following

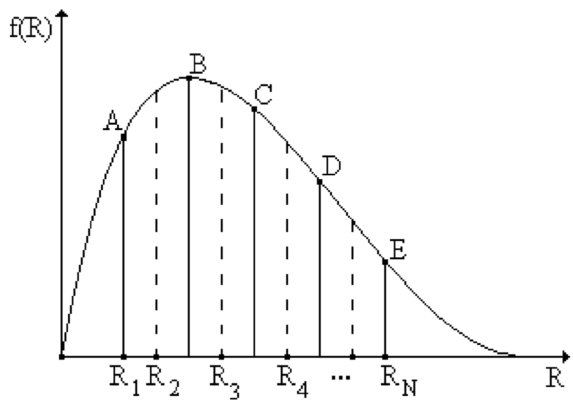

Fig. 3. Radius distribution function of droplets.

technique. The range of radii of the droplets is divided into N groups, where N is an arbitrary integer parameter. For each intermediate group we replace the actual distribution of droplets by a monodisperse one, with the radius being equal to the average radius for the given group of droplets. The total number of droplets in every group is constant. For two extreme groups, i.e., of the largest and smallest radii, we accept the value of the largest radius and of the smallest one. It is natural that the smaller the range of change of radius in the group, the more exact is the description of disperse phase behavior. These questions in more detail are considered in the Appendix.

The following physical assumptions are assumed in our model of evaporative cooling of droplets: droplets have a spherical shape; an approximation of average droplet temperature is used. Besides the semi-empirical dependences of the heat and mass transfer coefficients of a droplet in a gas flow and coefficient of aerodynamic drag, all depending on the Reynolds number for droplets.

Let us now describe the mathematical model of evaporative cooling of water droplets. We direct the z-axis vertically downward and fix the coordinate origin at the point of the beginning of droplet fall. The falling droplet experiences the action of the gravity force and force of aerodynamic drag, which determines the change in the velocity of droplets and their density per volume unit. As a rule, for small size for mechanical draft cooling towers velocities of droplets are increased monotonously during their fall. The system of the differential equations includes N equations that describe a change in the radii of droplets $R_i(z)$ due to evaporation:

$$\frac{dR_i(z)}{dz} = -\frac{\gamma(Re_i)[\rho_s(T_{wi}(z)) - \rho(z)]}{\rho_w v_i(z)} \tag{10}$$

and N equations that, determine a change in the velocities $v_i(z)$ of the falling droplets:

$$\frac{dv_i(z)}{dz} = \frac{g}{v_i(z)} - C(Re_i)\frac{\rho_a[v_i(z) - u(z)]^2}{2v_i(z)}\frac{\pi R_i(z)^2}{m_i}. \tag{11}$$

We note that allowance for the accelerated motion of droplets is of great importance for relatively small mechanical draft cooling towers, because the droplets has no time to reach the steady-state velocity. For large droplets, Eq. (11) can be solved at constant value of droplet radius, because the droplet radius changes less than 1% due to evaporation.

We have N equations, describing a change in the volume-averaged temperature of the droplets $T_{wi}(z)$:

$$\frac{dT_{wi}(z)}{dz} = \frac{3\{\alpha(Re_i)[T_a(z) - T_{wi}(z)] + (-\gamma(Re_i))(r - c_w T_{wi}(z))\lfloor\rho_s(T_{wi}(z)) - \rho(z)\rfloor\}}{c_w \rho_w R_i(z) v_i(z)}. \tag{12}$$

To calculating the change in the temperature of humid air $T_a(z)$, the equation has the form

$$\frac{dT_a(z)}{dz} = \frac{4\pi}{\rho_a c_a}\sum_{i=1}^{N}\frac{R_i(z)^2 N_{di}(z)}{(v_i(z) - u(z))}[\alpha(Re_i)[T_a(z) - T_{wi}(z)]]. \tag{13}$$

It is worth to note that the rate of change of air temperature is directly proportional to the total interfacial surface area, $\sum_{i=1}^{N} 4\pi R_i^2 N_{di}$, and is inversely proportional to the relative velocity of phases.

The equation that describes a change in the density of water vapor $\rho(z)$ in the air–vapor mixture is:

$$\frac{d\rho(z)}{dz} = -4\pi\sum_{i=1}^{N}\frac{R_i(z)^2 N_{di}(z)}{v_i(z) - u(z)}\gamma(Re_i)[\rho_s(T_{wi}(z)) - \rho(z)]. \tag{14}$$

The boundary conditions for the system of Eqs. (10)–(14) are:

At $z = 0$ (point of beginning of droplet fall) the following values are defined for:

droplets radii

$$R_i|_{z=0} = R_{i0}, \tag{15}$$

temperatures of droplets for each group

$$T_{wi}|_{z=0} = T_{wi0}, \tag{16}$$

initial velocities of droplets (for simplicity we consider them to have the same value)

$$v_i|_{z=0} = v_0. \tag{17}$$

At $z = H$:

the air temperature

$$T_a|_{z=H} = T_{a0}, \tag{18}$$

the density of the water vapor in the air

$$\rho|_{z=H} = \rho_0. \tag{19}$$

Thus, the system of ordinary differential equations (10)–(14) and boundary conditions (15)–(19) represent the nonlinear boundary-value problem.

Attention is to be drawn to the fact that in our model the influence of the number of droplets per unit volume on the parameters of humid air is taken into account. The number of droplets per unit volume, $N_{di}(z)$ is defined by the specific water flow rate Q_{wi}, the sizes R_i of droplets and their velocities v_i as

$$N_{di}(z) = \frac{3Q_{wi}}{4\rho_w \pi R_i^3 v_i(z)}. \tag{20}$$

It follows from Eq. (20) that the number of droplets per unit volume decreases with increase in velocities of droplets at a constant water flow rate. As a rule, for mechanical draft cooling towers [2,3] the hydraulic loads are such that the average distance between droplets is much greater than their diameter. This fact taken into account in our mathematical model by using the heat and mass transfer coefficients obtained for a single droplet. We define the average droplet radius as follows:

$$\langle R \rangle = \frac{\sum_{i=1}^{N} R_i Q_{wi}}{\sum_{i=1}^{N} Q_{wi}} = \frac{\sum_{i=1}^{N} R_i^4 N_{di}}{\sum_{i=1}^{N} R_i^3 N_{di}}. \tag{21}$$

Similarly, the formula for the average temperature of droplets is determined as

$$\langle T_w \rangle = \frac{\sum_{i=1}^{N} T_{wi} Q_{wi}}{\sum_{i=1}^{N} Q_{wi}} = \frac{\sum_{i=1}^{N} T_{wi} R_i^3 N_{di}}{\sum_{i=1}^{N} R_i^3 N_{di}}, \tag{22}$$

where Q_{wi} is the water mass flow rate for droplets of radius R_i.

In accordance with the results of [13], the coefficient of heat exchange of a droplet with the air medium, $\alpha(Re_i)$, was determined from the following dimensionless relation:

$$Nu = 2 + 0.5Re^{0.5}. \tag{23}$$

For droplets from the ith group, the Reynolds number is defined as

$$Re_i = \frac{2\rho_a R_i |v_i(z) - u(z)|}{\mu_a}, \tag{24}$$

where μ_a is the dynamic viscosity of air. The Nusselt number is calculated as $Nu = 2R_i\alpha(Re_i)/\lambda_a$.

Using the analogy between the heat and mass transfer coefficients, the coefficient of mass exchange $\gamma(Re_i)$ for a falling droplet with an ascending air flow is determined as

$$\gamma(Re_i) = \frac{D(2 + 0.5Re_i^{0.5})}{2R_i(z)}. \tag{25}$$

The coefficient of aerodynamic drag force of a droplet $C(Re_i)$, is calculated from formula [13]

$$C(Re_i) = \frac{24}{Re_i}\left(1 + \frac{1}{6}Re_i^{2/3}\right). \tag{26}$$

In numerical calculations we take into account the temperature dependence of the diffusion coefficient of water vapor in air, viscosity and thermal conductivity of air. The temperature dependence of the transfer coefficients was calculated according to [14].

In approximation of the average droplet radius [4], the results of comparison of our calculations with available experimental data, obtained the still air [15], are shown in Fig. 4. As it seen our model qualitatively correctly describes the cooling of the water droplet falling in air. The ratio of the calculated value of droplet temperature change $(\Delta T_w)_{theor}$ and experimental value $(\Delta T_w)_{exper}$ for different heights of droplet fall H is given in Table 1.

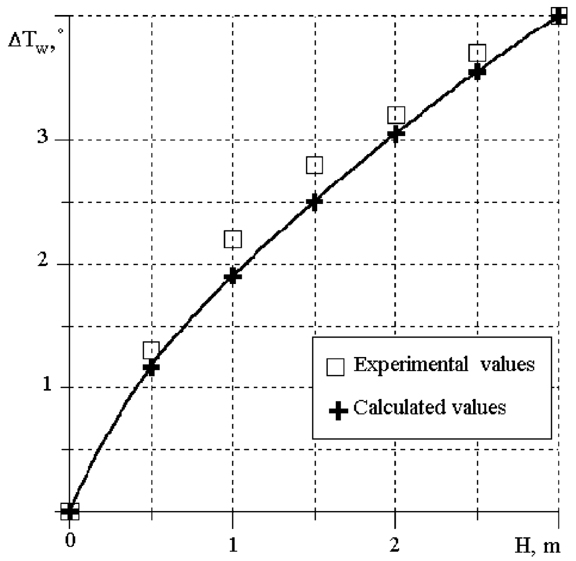

Fig. 4. Droplet temperature drop ΔT_w versus droplet fall height H: (□) experimental values [15] and (+) calculated values.

Table 1
Dependence of the ratio of calculated droplet temperature drop $(\Delta T_w)_{theor}$ and experimental drop $(\Delta T_w)_{exper}$ versus droplet fall height H

Droplet fall height H, m	0.5	1.5	3.0
$(\Delta T_w)_{theor}/(\Delta T_w)_{exper}$	0.9	0.9	1.0

As seen from Table 1, the mathematical model allows one to calculate the temperature of droplets. In fact, the ratio of the calculated value of droplet temperature difference $(\Delta T_w)_{theor}$ and the experimental value $(\Delta T_w)_{exper}$ for different heights of droplet fall H does not exceed 10%. Allowance for the weak free convection of air in the laboratory rig always existing during experimental investigation one increases the relative accuracy 3% even for small heights of fall.

4. Results of simulations

Before presenting numerical results, we give semi-quantitative estimations [12] of evaporative cooling of droplets in a mechanical draft cooling tower. These estimations are obtained by the approximate analytical integration of the system of equations described above.

To change the droplet temperature ΔT_w considering its accelerated motion, we have the qualitative estimation:

$$\Delta T_{wi} \sim \{\lambda_a[T_{a0} - T_{w0i}] + Dr[\rho_s(T_{w0}) - \rho_0]\}H^{0.5}u^{0.5}R_{0i}^{-3/2}. \quad (27)$$

It is interesting to note that the temperature drop ΔT_{wi} is directly proportional to $(Hu)^{0.5}$, which agrees well with the data of numerical results presented below. We emphasize that, as follows from (27), there is a strong inverse dependence of ΔT_{wi} on the initial droplet radius R_{0i}. After transformation of Eq. (27) the approximate expression for the thermal efficiency η of a cooling tower is:

$$\eta \sim \left\{\lambda_a \frac{[T_{a0} - T_{w0i}]}{(T_{w0i} - T_{lim})R_{0i}} + Dr\frac{[\rho_s(T_{w0i}) - \rho_0]}{(T_{w0i} - T_{lim})R_{0i}}\right\}\left(\frac{H}{R_{0i}}\right)^{0.5} u^{0.5}. \quad (28)$$

It is seen from Eq. (28) that for a mechanical draft cooling tower the thermal efficiency η depends on many parameters; with the contribution of the second element in Eq. (28) being the basic one. Since the diffusion coefficient $D \sim 1/P$, the thermal efficiency η depends on atmospheric pressure [9,16], as confirmed by our calculations.

In the approximation of the monodisperse ensemble of droplets of radius R, using qualitative description of evaporative cooling and the dimensionality theory [17], we have the formula

$$\eta = F(Q_w/Q_a, H/R, (T_{a0} - T_{w0})/(T_{w0} - T_{lim})). \quad (29)$$

The detailed analysis of expression (29) was carried out in [4]. In particular, it was shown that the first two arguments in Eq. (29) play the basic role in description of the thermal efficiency η of the considered type of mechanical draft cooling towers. It is worth noting that formula (29), obtained on the grounds of the dimensionality theory, enables one to substantially reduce the number of computing experiments.

In computer experiments, the boundary-value problem of evaporative cooling of droplets (10)–(19) was solved by the "shooting method" [18]. To obtain numerical solution of the system of differential equations, the Runge–Kutta method of the fourth order was used. The accuracy was checked by means of the residual criterion Σ:

$$\Sigma(T_{a0}, \rho_0) = \sqrt{\left(\frac{T_a(H) - T_{a0}}{T_{a0}}\right)^2 + \left(\frac{\rho(H) - \rho_0}{\rho_0}\right)^2}, \quad (30)$$

where $T_a(H)$ and; $\rho(H)$ are the results of calculation of the boundary-value problem (10)–(19). The solution of the problem terminated as soon as the condition $\Sigma < 10^{-4}$ was fulfilled.

Following [12,13], it is possible to show that the characteristic time of temperature relaxation inside the droplet τ_d is connected with the convective flow of liquid inside the droplet. This time can be estimated from the formula

$$\tau_d \sim \frac{R}{u}\sqrt{\rho_a/\rho_w}.$$

In particular, for a droplet of radius 1 mm and an air flow velocity of 2 m/s, $\tau_d \approx 10^{-2}$ s. Thus, the approximation of average droplet temperature is good enough when investigating of evaporative cooling of water droplets. This circumstance was used at development of our mathematical model.

The calculated dependence of the thermal efficiency η versus the dimensionless parameter H/R is shown in Fig. 5. It is seen that the dependence of the thermal efficiency of a cooling tower on the height of droplet fall is nonlinear, which qualitatively corresponds to Eq. (28). The effect of saturation arising because of the increase in humidity and temperature of humid air with increase in the parameter H/R is well seen. An increase in the velocities of droplets with increase in their height of fall also plays an important role, since the time of interaction of droplets with a cold air is reduced.

The dependence of the thermal efficiency η on the ratio between the mass flow rates of water and air Q_w/Q_a is shown in Fig. 6. As this ratio increases, the thermal efficiency of a cooling tower decreases. This is typical of all cooling towers [9]. For mechanical draft cooling towers the performance even at small values of the ratio

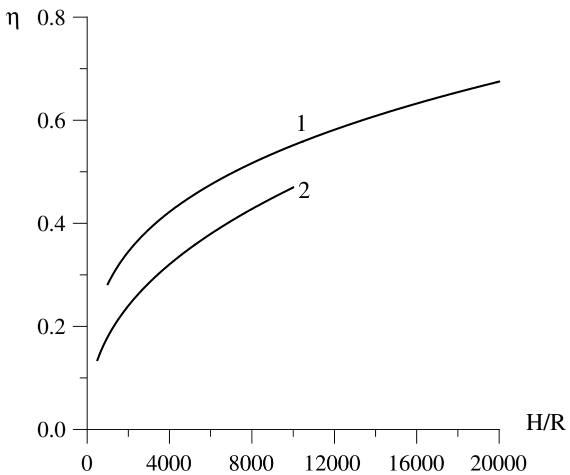

Fig. 5. Thermal efficiency η vs. H/R: curve 1 is for droplet radius $R = 0.5$ mm and curve 2 is for droplet radius $R = 1$ mm.

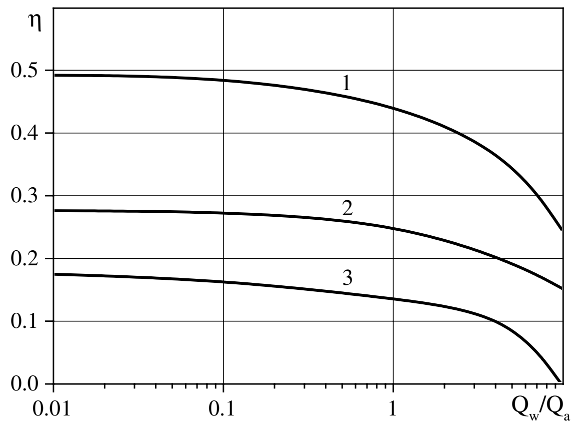

Fig. 6. Thermal efficiency η vs. Q_w/Q_a: curve 1 is for droplet radius $R = 0.5$ mm, curve 2 is for droplet radius $R = 1$ mm and curve 3 is for $R = 1.5$ mm.

Q_w/Q_a is possible, which corresponds to small hydraulic load or large air flow rate through the cooling tower. The dependence of η on the radius of droplets is rather strong and qualitatively corresponds to Eqs. (27) and (28). It is interesting to note that the partial allowance for two-dimensional aerodynamic effects, in accordance with Eq. (8), practically does not change the average value of the cooling tower thermal efficiency. Probably, this fact can be explained by the integral nature of this parameter.

The temperature profile of humid air for different heights in cooling tower for various droplets radii is displayed in Fig. 7. The droplets with rather small radii rapidly cool down to $T_{\lim}$ and, as a result, cool air to the temperature, which is lower than the temperature of surrounding air. This effect was experimentally verified in [19] and found interesting technical applications in power engineering [20]. These curves also illustrate the fact that the temperature profile of humid air depends on the distribution function of droplets; for large enough droplets this dependence has practically exponentional character.

The change of the air relative humidity inside the cooling tower is shown in Fig. 8. The air humidity monotonically increases and reaches its maximum at the cooling tower exit. The smaller size of droplets, the more intensive are the processes of heat and mass transfer. Therefore, a constant water flow rate as in Fig. 8, for droplets with smaller radii the humidity increases more rapidly.

For the constant water flow rate and the number groups of droplets $N = 3$, the influence of polydispersion on the thermal efficiency of a cooling tower is shown in Table 2. Due to the constant of the water flow rate only two from three parameters N_0, N_1, and N_2 are independent. The values for Table 2 were obtained for $Q_w = 2.6$ kg/m^2 s, $u = 2$ m/s, $H = 2$ m. In the first two columns of Table 2 the ensemble-average radius is equal to 1.2 and 1.4 mm, respectively; in the remaining columns the average radius is equal to 1 mm. It is seen that at the same average radius the polydispersity effects alter the value

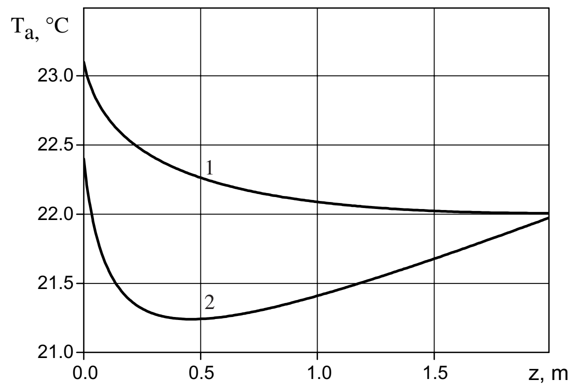

Fig. 7. Moist air temperature profiles in a cooling tower: curve 1 is for $R = 1$ mm and curve 2 is for $R = 0.5$ mm.

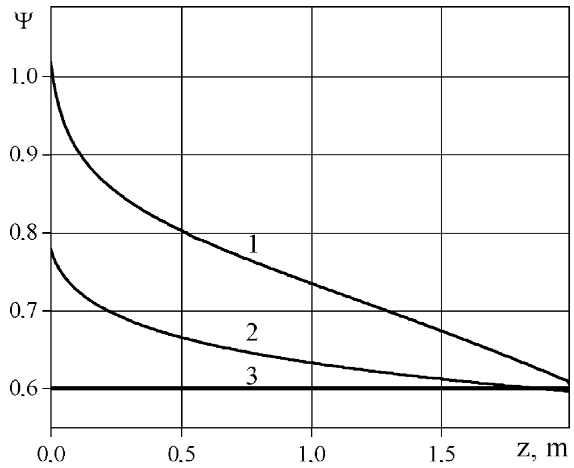

Fig. 8. Relative humidity of air in a cooling tower: curve 1 is for droplet radius $R = 0.5$ mm, curve 2 is for droplet radius $R = 1$ mm and curve 3 is for relative humidity of entering air.

Table 2
Thermal efficiency for different values of droplet number at different ranges

Number of droplets, $N_0 \times 10^{-5}$	5	9	0.7	49	9.7
Number of droplets, $N_1 \times 10^{-5}$	5	0.1	12	0.1	9.7
Number of droplets, $N_2 \times 10^{-5}$	2	3.3	0.097	1.8	0.45
η, %	57	51	66	68	66

of the average thermal efficiency by ±1%. It is natural that with decrease in the height of a cooling tower the influence of polydispersity increases, because only small droplets have time to be cooled. Similarly, as the hydraulic load increases the relative influence of the droplets distribution function increases.

Since droplets of different radii cool down with different rates, water temperature fluctuations must be observed on the pool surface. Indeed, because of the casual nature of the space distribution of droplets, they appear randomly in the fixed region of the pool surface. Conducting measurements experimentalists note water temperature fluctuations in the cooling tower pool [21]. As a measure of these fluctuations, we have calculated the dispersion of water temperature Dt from of the equation

$$Dt = \frac{\sum_{i=1}^{N} (T_{wi} - \langle T_w \rangle)^2 \cdot Q_{wi}}{\sum_{i=1}^{N} Q_{wi}}.$$

For illustration, Table 3 presents calculated values of the dispersion of water temperature for the conditions given in Table 2. It is seen that when the radius distribution function of droplet is rather wide (the 4th column) the temperature dispersion is rather large. For a narrow distribution function, the dispersion is sharply reduced. Thus, for different number of droplets in groups, but at same average radius we have rather close values of cooling tower thermal efficiency, however, dispersions of the final temperature of droplets can be rather different.

In the approximation of an average radius, we estimate dissipation W of the kinetic energy of an air flow, caused by friction of droplets against the ascending flow. The specific dissipation W is directly proportional to the specific mass water flow rate and, naturally depends the radius on droplets and their height of fall. It can be presented as the difference of the energy fluxes:

$$W = Q_w \frac{\left(v_0(R,H)^2 - v_1(R,H)^2\right)}{2}, \quad (31)$$

where v_0 (R,H) and v_1 (R,H) are the velocity of water droplets of radius R after their fall from the height H in

Table 3
Droplets temperature dispersion for different functions of distribution

Number of droplets, $N_0 \times 10^{-5}$	5	9	0.7	49	9.7
Number of droplets, $N_1 \times 10^{-5}$	5	0.1	12	0.1	9.7
Number of droplets, $N_2 \times 10^{-5}$	2	3.3	0.097	1.8	0.45
Water temperature dispersion, Dt (°C)^2	11.9	11.9	1.1	35.8	7.8

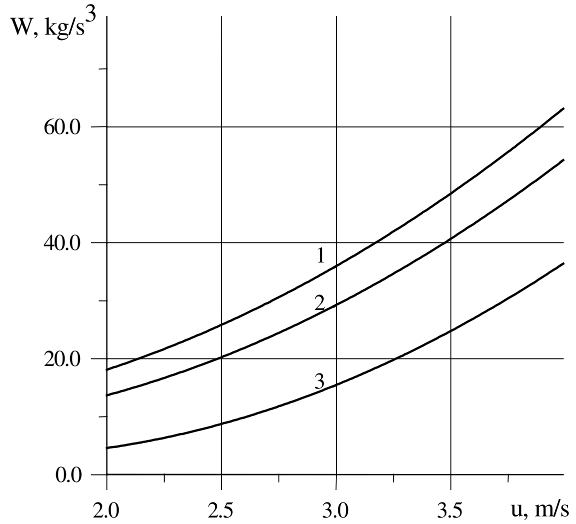

Fig. 9. Specific flow power dissipation W vs. its velocity u. The dependence is obtained for $H = 2$ m, $R_0 = 1$ mm. Curve 1 is for $Q_w = 2.6$ kg/m^{2}s, curve 2 is for $Q_w = 1.7$ kg/m^{2}s and curve 3 is for no rain resistance (W (J m/s) and u (m/s)).

still air, and the velocity of the same droplets for the same height of fall H, but under the condition of ascending air flow with the velocity u. For the cooling tower height $H = 2$ m, for initial droplets of radius $R = 1$ mm and for two values of the specific water mass flow rates $Q_w = 1.7$ and 2.6 kg/m^2 s, the dependence of the specific dissipation W on the air velocity is shown in Fig. 9. It is seen, that for the cooling tower performance in a given aerodynamic mode, "the rain resistance" requires substantial enhancement of the fan power, and the higher the cooling tower, the greater is the contribution of this resistance.

For a steady-state regime of the performance of a mechanical draft cooling tower, the developed mathematical model of evaporative cooling gives the flow rate of evaporated water. We note that the higher the velocity of air flow in the cooling tower, the larger its efficiency and larger flow rate of evaporated water. For example, when the air velocity increases from $u = 2$ m/s up to 4 m/s, the thermal efficiency of the cooling tower increases from 25% up to 33% (at the fixed radius $R = 1$ mm, $T_{w0} = 40.7$ °C, $T_{a0} = 23.4$ °C, $\psi = 0.36$, $Q_w = 2.6$ kg/m^2 s). In turn, the flow rate of evaporated water increases from 1.5% up to 3% of the flow rate of the water in the cooling tower.

5. Optimization of the cooling tower performance

Optimization of the cooling tower performance is one of the most important problems in the theory and engineering practice of evaporative cooling of water. Some aspects of this problem are discussed in [22,23], however it is necessary to carry out still great number of theoretical and experimental investigations dealing with creation of a control system for a cooling tower. In this section we consider optimization of the performance of a mechanical draft cooling tower [24] in the following formulation of the problem: it is required to determine the minimal air flow rate through the cooling tower (air flow rate was increased discretely with a given step) for reaching the given constant temperature T_1 of the water which leaving the cooling tower. Moreover, the initial circulating water temperature and its flow rate are considered constant, and the temperature and humidity of air are variable quantities. Thus formulation of the optimization problem reflects the practical problem of maintenance of the thermal performance of some technological installation.

For determining the minimal air flow rate, the developed mathematical model (10)–(19) was solved by an iterative method. At the given polydispersity of water the problem of evaporative cooling was solved at an arbitrary initial air flow rate. If the final temperature of water T_w is higher than the given value of temperature T_1, the air flow rate is increased discretely at a given step. This process was repeated until the condition $T_w \leqslant T_1$ was met. Some of the calculation results for this optimization problem are presented in Fig. 10 for changing temperature and humidity of the air surrounding the cooling tower. The temperature T_1 was equal to 22 °C; and the air temperature was higher, equal to or lower than T_1. In the latter case, the role of evaporative cooling is especially great. As it seen from Fig. 10, with increase in the humidity of the atmospheric air it is necessary to increase the air flow rate through cooling tower. The higher the air temperature, the larger should be tangent of the angle of the inclination of curves. This is connected with the increasing of the role of convective heat transfer. Note that the curve corresponding to $T_a = 22$ °C ends when relative humidity is 0.65. This is due to the fact that at large humidity the air flow rate becomes so large that the droplets are entrained blow out by the ascending air flow. This leads to a loss of water in a cooling tower.

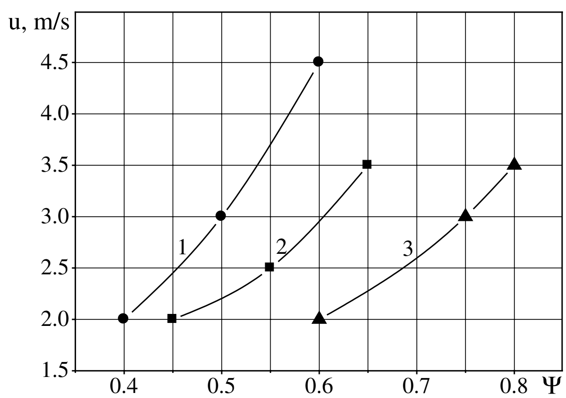

Fig. 10. Optimal air velocity vs. its humidity: curve 1 is for air temperature 25 °C, curve 2 is for air temperature 22 °C and curve 3 is for air temperature 20 °C.

6. Discussion of results

For the counter-flow mechanical draft cooling tower a one-dimensional mathematical model has been developed. It represents the boundary-value problem for a system of ordinary nonlinear differential equations, describing interrelated heat and mass transfer processes and the dynamics of fall of droplets. Moreover, our model includes a distribution function of droplet radii. The methods of solution of such problem is proposed, computer code is created and numerical simulation is made.

Qualitative estimations for thermal efficiency of spray mechanical draft cooling tower are presented. In particular, it was shown that the thermal efficiency of the cooling tower η depends essentially on the ratio H/R.

For a monodisperse ensemble of droplets in a mechanical draft cooling tower, the limits of applicability of this approximation for the description of evaporative cooling are determined. As a result of numerical experiments it is demonstrated that the average thermal efficiency depends practically only on the third moment of the distribution function $\langle R^3 \rangle$. Relative deviations of the thermal efficiency can reach several percent. This conclusion makes the results of [25] more specific. It was found that a variety of effects cannot be described well in approximation of monodisperse ensemble of droplets, and, in particular, the air temperature profile and water temperature fluctuations on the surface of a water-collecting pond.

The dependence of the thermal efficiency of a mechanical draft cooling tower on the ratio between the mass flow rates of water and air is determined. For mechanical draft cooling towers, this dependence is weaker than for other types of cooling towers. For small values Q_w/Q_a, when they change from 0.01 to 1, the thermal efficiency of the cooling tower practically does not change.

The flow rate of evaporated water has been reckoned in the steady-state regime and it is shown that it can reach 3% of the mass flow rate of water; with this value depending on atmospheric pressure. The evaporation is stronger at lower atmospheric pressure as diffusion coefficient of the water vapor increases.

The mathematical model of a control system for a mechanical draft cooling tower at varying parameters of air is developed. This model allows optimization of the cooling tower performance adjusting a fan power to changing atmosphere conditions.

A method is developed for calculating of the dissipation of kinetic energy due to the friction of falling droplets with an ascending air flow. In particular, it is shown that the higher the cooling towers, the more substantial is this dissipation. Predictability of our mathematical model can be enhanced by including two-dimensional aerodynamic effects in a mechanical draft cooling tower. The work in this direction is being carried out.

Mathematical model of mechanical draft cooling towers with film flows of water is being now developed based on the approach of [8]. The optimization of geometric parameters and design of such cooling towers is a vital problem for energy saving in industry, air conditioning, etc.

Appendix A

In cooling towers the droplet radii distribution function $f(R)$ is known very approximately as it depends on many badly controlled parameters. Below, we consider the problem of determining this distribution function of droplets by means of the concept of the maximum of the information entropy [26,27].

We define the distribution function $f(R)$ per unit of the volume whose normalization is equal to unity

$$\int_0^\infty f(R)\,\mathrm{d}R = 1, \tag{A.1}$$

then the mass flow rate of water Q_w per unit of the cross-section can be presented in the form

$$\frac{4\pi N_d v}{3}\int_0^\infty f(R)R^3\,\mathrm{d}R = Q_w, \tag{A.2}$$

where v is the droplets velocity, N_d is the number of droplets per unit volume. Moreover, we consider that the initial velocities of all droplets are identical.

The distribution function has an important property, namely

$$f(0) = 0. \tag{A.3}$$

For our problem, following [26,27], we enter the information entropy S of the system under consideration as

$$S = -\int_0^\infty f(R)\ln f(R)\,\mathrm{d}R. \tag{A.4}$$

Such distribution functions, for which the information entropy is extreme, more precisely describe experimental situations with a shortage of information.

We will seek the distribution function of droplet radius from the condition of maximum of information entropy (A.4) under condition of the conservation of normalization condition (A.1), the mass flow rate of water (A.2), and the average logarithm of droplets radii. The average logarithm of droplets radii is defined as

$$\int_0^\infty \ln R\cdot f(R)\,\mathrm{d}R = \mathrm{const.} \tag{A.5}$$

As it is shown below, Eq. (A.5) ensures the fulfillment of expression (A.3). From the condition of maximum of S and the fulfillment of Eqs. (A.1), (A.2) and (A.5), we have expression for $f(R)$:

$$f(R) = \frac{1}{Z}R^\delta\exp(-\beta R^3), \tag{A.6}$$

where

$$Z = \int_0^\infty R^\delta\exp(-\beta R^3)\,\mathrm{d}R. \tag{A.7}$$

The Lagrange multipliers δ and β are determined from of Eqs. (A.2) and (A.5). Our result differs from similar one presented in [28].

Since the constant in (A.5) is unknown, the parameter δ should be selected based on other considerations. In particular, for existence of normalization of the distribution function of droplet sizes, the parameter δ should satisfy the condition $\delta \geqslant 2$. Then, the form of distribution function (A.6) qualitatively coincides with the graph in Fig. 3. Further we accept, that the value $\delta = 2$. If the additional information about other moments of the distribution function of droplets is known, then it is possible to find the type of the size distribution function of droplets taking into account this information.

If the parameter δ in expression (A.6) is equal to 2, then parameter β is easily determined from condition (A.2). After integration we have

$$\beta = \frac{4\pi N_d v\rho_w}{3Q_w} = \frac{1}{\langle R^3\rangle},$$

where $\langle R^3\rangle$ is the average cube of radii of the droplets. Thus the distribution function $f(R)$ can be presented in the form

$$f(R) = \frac{3R^2}{\langle R^3\rangle}\exp\left[-\frac{R^3}{\langle R^3\rangle}\right], \tag{A.8}$$

it is one-parametrical Weibull distribution [29]. The maximum value of the distribution function is reached at the radius value $R = 2(\langle R^3\rangle)^{1/3}/3$. Thus, the average radius is equal to $0.893\langle R^3\rangle^{1/3}$, and the average square of the radius of droplets is equal to $0.9\langle R^3\rangle^{2/3}$.

For distribution function (A.8), we demonstrate our method of discretization of the distribution function of droplet radii. For simplicity we assume that $\langle R^3\rangle = 1$ mm, $v = 0.5$ m/s, $Q_w = 2.6$ kg/m^2 s. For $N = 3$, the number of droplets in each of the ranges of division is shown in Table 4. The total number of droplets of all

Table 4
Fraction of droplets for different ranges of radii vary

Range of separation, mm	0–0.5	0.5–1.5	1.5–3
Relative fraction of droplets in given range	0.118	0.848	0.034

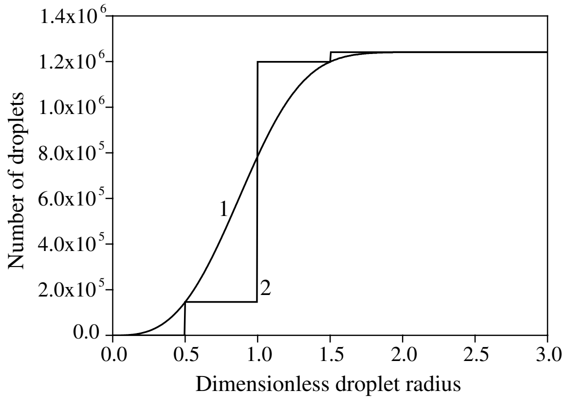

Fig. 11. The number of droplets of given radius for the Weibull distribution and its discretization. Curve 1 is based on Weibull distribution function, curve 2 is an application of an discretization method.

sizes is equal to $N_d = 1.241 \times 10^6$. Calculation based on our mathematical model gives the value of heat efficiency $\eta = 65\%$ and $Dt = 4.1$, which practically coincides with the data presented in Table 2. For illustration, in Fig. 11 the number of droplets with the radii, smaller than the given radius or equal to it, is shown for distribution function of droplet radius (A.8). These values are also given for our method of the discretization of the distribution function. For the discretization parameter $N = 3$, the characteristic features of the size distribution function of droplet are transmitted not exactly. However, for the description of such integrated parameter as the cooling tower thermal efficiency, the achieved accuracy is sufficient.

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